Math 131A: Analysis
Discussion 1: Introduction to Proofs

1 Direct Proofs

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1. Prove for all $x, y \in \mathbb{Q}$, $x + y \in \mathbb{Q}$ and $xy \in \mathbb{Q}$.
2. If n is an odd integer, prove that n^2 is an odd integer.
3. Prove for all $x, y \in \mathbb{R}$, $|x + y| \leq |x| + |y|$.

2 Proof by Contradiction

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1. Prove that $\sqrt{2}$ is irrational.
2. Prove that a rational number plus an irrational number must be irrational.
3. **Challenge:** Prove there are infinitely many prime numbers.

3 Proof by Contrapositive

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1. If n is an integer and n^2 is odd, prove that n is odd.
2. Suppose $x, y \in \mathbb{Z}$. If $5 \nmid xy$, then $5 \nmid x$ and $5 \nmid y$.
3. Let $x \in \mathbb{Z}$. If $x^2 - 6x + 5$ is even, then x is odd.