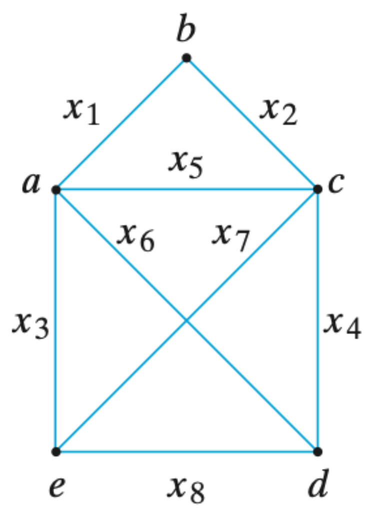
Math 61: Introduction to Discrete Structures
Discussion 10: Adjacency Matrices and Trees

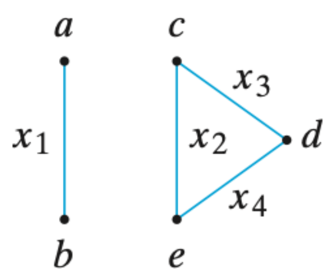
## 1 Adjacency Matrices

*In Exercises 1–6, write the adjacency matrix of each graph.*

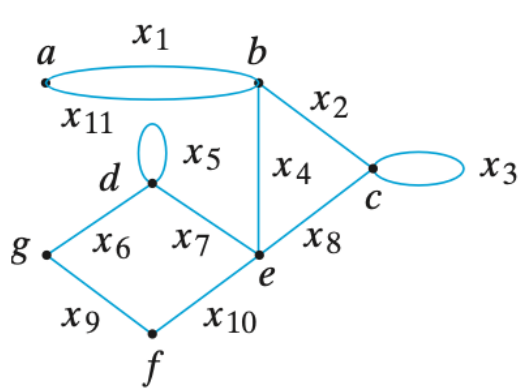
**1.**




**2.**




**3.**



**24.** $\begin{array}{c} a \\ b \\ c \\ d \\ e \end{array} \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 \end{pmatrix}$ **25.** $\begin{array}{c} a \\ b \\ c \\ d \\ e \end{array} \begin{pmatrix} 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}$

**26.** What must a graph look like if some row of its incidence matrix consists only of 0's?

1. Let $A$ be the adjacency matrix of an undirected graph $G$. Prove (using recurrence relations or induction) that the $(i, j)$-th entry of $A^n$ counts the number of paths from $v_i$ to $v_j$ in $G$.

## 2 Trees

1. Recall that a *forest* is a simple graph with no cycles. Explain why a forest is a union of trees.
2. If a forest consists of $m$ trees and $n$ vertices, how many edges does it have?
3. Show that a graph $G$ with $n$ vertices and fewer than $n-1$ edges is not connected.
4. Recall that an *articulation point* is a vertex which, when removed, causes the graph to become disconnected (increases number of components).
   (a) Show if $G$ is a tree, every vertex of degree greater than 1 is an articulation point.
   (b) Give an example to show the converse is false (every vertex of degree greater than 1 is an articulation point, but $G$ is not a tree), even if $G$ is assumed to be connected.

## 3 Planar Graphs

1. Show that any graph with at most 4 vertices is planar (without appealing to Kuratowski's Theorem).
2. Show that a simple connected planar graph with at least two faces satisfies the inequality $e \leq 3v - 6$.