Math 61: Introduction to Discrete Structures
Discussion 2: Functions (cont.) and Intro to Proofs (Section 2.1)

1 Functions (cont.)

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1. How many functions are there from $\{1,2\}$ to $\{a,b\}$? Which ones are onto? Which ones are one-to-one?
2. Determine whether the following functions $f : \mathbb{Z} \to \mathbb{Z}$ are one-to-one or onto, or both. Prove your answers
   - (i) $f(n) = n+1$
   - (ii) $f(n) = 2n$
   - (iii) $f(n) = n^2 - 1$
   - (iv) $f(n) = n^3$
3. Let $f : X \to Y$ be a function. Let
$$\mathcal{S} = \{f^{-1}(\{y\}) : y \in Y\}$$
Show $\mathcal{S}$ is a partition of X.
4. Let f and g be functions from $\mathbb{R}_{\geq 0}$ to $\mathbb{R}_{\geq 0}$ defined by the equations
$$f(x) = \lfloor 2x \rfloor, \quad g(x) = x^2$$
Find the compositions $f \circ f, g \circ g, f \circ g, g \circ f$.
5. Given
$$f = \{(a,b),(b,a),(c,b)\},$$
a function from $\{a,b,c\}$ to $\{a,b,c\}$.
   - (a) Write $f \circ f$ and $f \circ f \circ f$ as sets of ordered pairs
   - (b) Define $f^n = f \circ f \circ \ldots \circ f$ (n-times). Write f^9 and f^{623} as sets of ordered pairs.
6. Prove that if n is an odd integer, then
$$\lfloor \frac{n^2}{4} \rfloor = \left(\frac{n-1}{2}\right)\left(\frac{n+1}{2}\right)$$

2 Introduction to Proofs (Section 2.1)

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1. Prove that for all numbers $x, y \in \mathbb{Q}$, $x + y \in \mathbb{Q}$ and $xy \in \mathbb{Q}$.
2. If $\mathcal{P}(X) \subseteq \mathcal{P}(Y)$, then $X \subseteq Y$ for all sets X and Y.
3. Disprove that $\mathcal{P}(X \cup Y) \subseteq \mathcal{P}(X) \cup \mathcal{P}(Y)$ for all sets X and Y.
4. If a and b are real numbers we define $\max\{a, b\}$ to be the maximum of a and b or the common value if they are equal. Prove that for all real numbers d, d_1, d_2, x,

$$\text{if } d = \max\{d_1, d_2\} \text{ and } x \geq d, \text{ then } x \geq d_1 \text{ and } x \geq d_2$$

5. Assume the following are given. Let $a, b, c \in \mathbb{R}$.
    - (Additive Identity for 0) $b + 0 = b$
    - (Distributive Law) $a(b + c) = ab + ac$
    - (Additive Cancellation) If $a + b = a + c$, then $b = c$

    Prove from these assumptions that $x \cdot 0 = 0$ for any $x \in \mathbb{R}$.