

Math 61: Introduction to Discrete Structures

Discussion 7: Recurrence Relations and Graphs

1 Pigeonhole Principle

1. Terrence is paid biweekly to teach Math 61. Show there exists a month in which he is paid three times.
2. Draw 6 integers from the set $\{1, 2, \dots, 10\}$. Show at least two of these chosen integers must sum to 11.

2 Recurrence Relations

1. Let P_n denote the number of permutations of n distinct objects. Find a recurrence relation and an initial condition for the sequence $\{P_i\}_{i=1}^{\infty}$.
2. Suppose we have n dollars and each day we buy either orange juice (\$1), milk (\$2), or soda (\$2). Find a recurrence relation for the ways we can spend this money. (The order in which we purchase items matters).
3. Let R_n denote the number of regions n lines divides the plane into, provided that no two lines are parallel, and no three lines meet at a point.
 - (a) Derive a recurrence relation for $\{R_i\}_{i=1}^{\infty}$.
 - (b) Solve the above recurrence relation.
4. Which of the following are linear homogeneous recurrence relations with constant coefficients? If it is, identify the order of the relation.
 - (i) $a_n = -3a_{n-1}$
 - (ii) $a_n = 2na_{n-2} - a_{n-1}$
 - (iii) $a_n = a_{n-1} + 1 + 2^n$
 - (iv) $a_n = -a_{n-1} + 5a_{n-2} - 3a_{n-3}$
 - (v) $a_n = 7a_{n-2} - 6a_{n-3}$
5. Solve the following recurrence relations
 - (i) $a_n = -3a_{n-1}, a_0 = 2$
 - (ii) $a_n = 7a_{n-1} - 10a_{n-2}, a_0 = 5, a_1 = 16$
 - (iii) $a_n = a_{n-1} + 1 + 2^n, a_0 = 0$

3 Graph Theory

1. Does there exist a graph G such that
 - (i) G has four edges, and four vertices with degrees 1, 2, 3, 4.
 - (ii) G has four vertices with degrees 1, 2, 3, 4.
 - (iii) G has five vertices, all with degree 3.
 - (iv) G is a simple graph with five vertices and degrees 2, 2, 4, 4, 4
2. Show that the maximum number of edges in a simple, disconnected graph with n vertices is $\frac{(n-1)(n-2)}{2}$
3. Let G be a connected graph. Suppose that an edge e is part of some cycle. Show that $G - \{e\}$ is still connected.